Pen and Paper Arguments for SIMON and SIMON-like Designs

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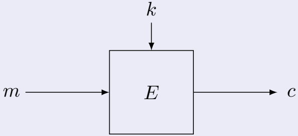
Block Ciphers

Definition

A block cipher is a function $E : \mathbb{F}_2^n \times \mathbb{F}_2^s \to \mathbb{F}_2^n$, such that $E(\cdot, k)$ is a permutation for every key $k \in \mathbb{F}_2^s$.



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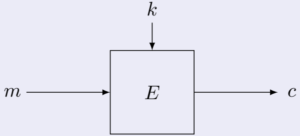
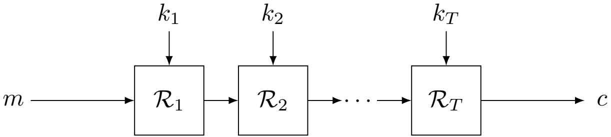


Typically, we use round-iterated constructions.



New Block Cipher Designs

- In the last years, many new primitives were proposed (e.g. CAESAR competition, lightweight designs)
- Lots of them use well-known constructions (e.g. AES-like ciphers)
- Some of them are more innovative (e.g. SIMON and SPECK)

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Common Sense: Explain your design!

New block ciphers should be designed in a way that allow for arguments on their security. Designers are expected to provide security arguments againt the most common attacks!

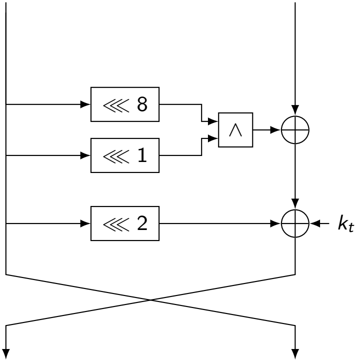
What is SIMON

- family of lightweight block ciphers designed for several block sizes and key length (10 versions in total)
- published by NSA in June 2013 on the IACR eprint archive[1]
- very simple and innovative construction

[1] R. Beaulieu et al. *The SIMON and SPECK Families of Lightweight Block Ciphers*. Cryptology ePrint Archive, Report 2013/404. `http://eprint.iacr.org/2013/404`. 2013.

Description of SIMON

- Feistel design
- A variety of block length supported (32, 48, 64, 96, 128 bit)
- The key length differs between 64 and 256 bit
- Simple round function
- 32 up to 72 rounds



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- Unfortunately, the designers of SIMON presented no design rationale of their ciphers.
- Lots of third-party analysis of SIMON was published. Most of the analysis is experimental.

Contribution

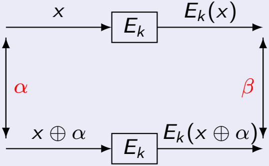
In this work, we focus on differential cryptanalysis.

- Considering differential attacks, we provide a non-experimental (pen and paper) security argument over multiple rounds of SIMON
- Thus, we contribute towards a better understanding of possible block cipher constructions.

Differential Cryptanalysis

Idea

For a function $E_k : \mathbb{F}_2^n \to \mathbb{F}_2^n$, we would like to consider a differential $\alpha \xrightarrow{E_k} \beta$.



The probability of a differential $\alpha \xrightarrow{E_k} \beta$ can be computed as

$$P(\alpha \xrightarrow{E_k} \beta) = \frac{\{x \in \mathbb{F}_2^n \mid \beta = E_k(x) \oplus E_k(x \oplus \alpha)\}}{2^n}.$$

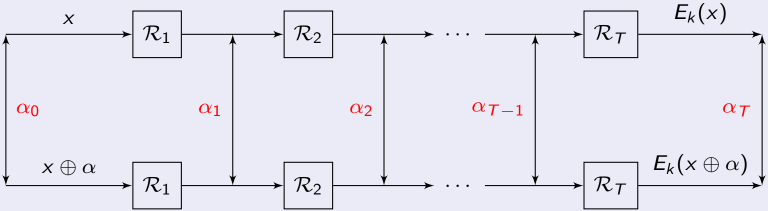
If E_k is a (round reduced) instance of a block cipher, the knowledge of a differential with high probability can be used as a distinguisher.

Considering Differential Trails

Usually, it is hard to compute the probability of multi-round differentials.

We consider differential trails

Let $\mathcal{R}_i$ denote the i-th round of a round-iterated cipher E_k. A T-round differential trail is a $(T+1)$-tuple of differential states.



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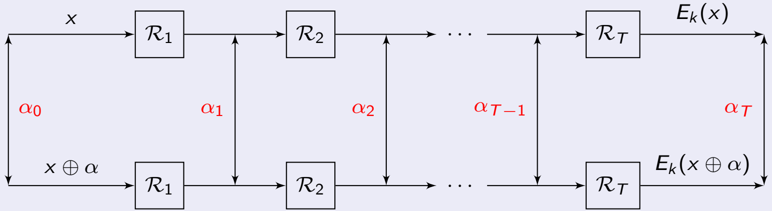

For round-iterated ciphers, we assume that the probability of a trail is the product of its single-round differentials. Thus,

$$P(\alpha_0 \xrightarrow{\mathcal{R}_1} \alpha_1 \xrightarrow{\mathcal{R}_2} \dots \xrightarrow{\mathcal{R}_T} \alpha_T) = \prod_{i=1}^{T} P(\alpha_{i-1} \xrightarrow{\mathcal{R}_i} \alpha_i).$$

Considering Differential Trails (cont.)

Common Security Argument

- Prove an upper bound on the max. probability of any differential trail over a certain number of rounds t. (typically $\leq 2^{-\text{blocksize}}$)
- Specify the number of rounds of the primitive as $t + \kappa$ for a reasonable security margin κ.

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Two common mehtods to prove such an upper bound

- Experimental search (e.g. MILP, SAT/SMT solver): Works quite well for word-based ciphers (SPNs) and bit-based ciphers (like SIMON)
- Pen and paper proof: Works well for AES-like ciphers (Wide-trail strategy[a])

[a] J. Daemen. "Cipher and hash function design strategies based on linear and differential cryptanalysis". PhD thesis. Doctoral Dissertation, March 1995, KU Leuven, 1995.

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Can we find more pen and paper arguments?

Results

- Considering differential attacks, we provide a non-experimental security argument over multiple rounds of SIMON.
- In particular, we bound the probability of t-round differential trails below 2^{-2t+2}.

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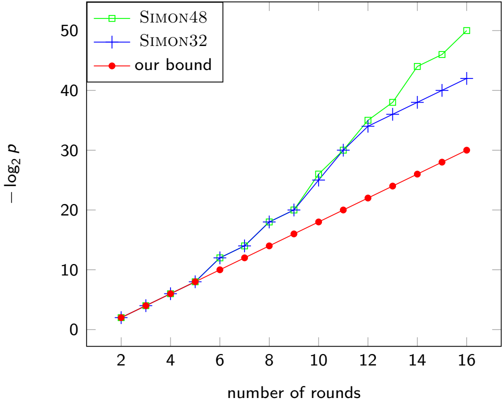
- In particular, we bound the probability of t-round differential trails below 2^{-2t+2}.

- Although our bounds are (much) worse than the best experimental bounds known, our argument shows that no attack based on a single differential trail is possible for all instances of SIMON.

Results (cont.)



Comparison of the experimental bounds[2] for SIMON32 and SIMON48 and our provable bounds.

[2] S. Kölbl et al. *Observations on the SIMON Block Cipher Family*. CRYPTO 2015.

Results (cont.)

Rounds needed for bounding the differential probability by $2^{-\text{blocksize}}$

	rounds	**rounds needed**	**margin**
SIMON 32/ 64	32	17	15
SIMON 48/ 72	36	25	11
SIMON 48/ 96	36	25	11
SIMON 64/ 96	42	33	9
SIMON 64/128	44	33	11
SIMON 96/ 96	52	49	3
SIMON 96/144	54	49	5
SIMON128/128	68	65	3
SIMON128/192	69	65	4
SIMON128/256	72	65	7

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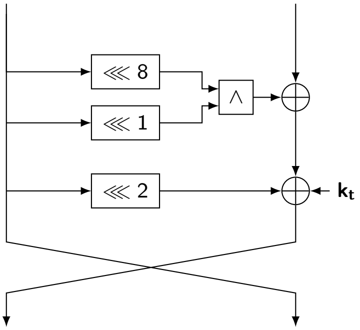
SIMON: linear and non-linear layer



- We seperate the Feistel function of SIMON into a non-linear part ρ and a linear part θ.

Our main result

Let $f_S(x) := (x \ggg 8) \wedge (x \ggg 1) \oplus (x \ggg 2)$ be the Feistel f-function.

Differential probability of SIMON

The probability of any t-round differential trail is upper bounded by 2^{-2t+2}.

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Differential probability of SIMON

The probability of any t-round differential trail is upper bounded by 2^{-2t+2}.

The main idea of the proof:

1. Show that the differential probability is low for input differences with large Hamming Weight (≥ 4)
2. Prove all other cases seperately

Some observations on the round function

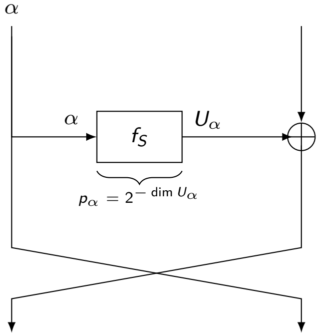
Let $f_S(x) := (x \lll 8) \wedge (x \lll 1) \oplus (x \lll 2)$

The single-round behavior is understood quite well.

Single-round propagation (Kölbl, Leander, Tiessen, 2015)

For a given (non-zero) input difference $\alpha \in \mathbb{F}_2^n$ into f_S, the set of possible output differences defines an affine subspace U_α s.t. $p_\alpha := P(\alpha \xrightarrow{f_S} \beta) \neq 0$ for all $\beta \in U_\alpha$. In particular, $p_\alpha = 2^{-d_\alpha}$ with $d_\alpha = \dim U_\alpha$.

Some observations on the round function



Why?
Because $\deg f_S = 2$ and thus $f_S(x) \oplus f_S(x \oplus \alpha)$ is linear

Some observations on the round function

$\Longrightarrow$ Observation: dim U_α (and thus the differential probability) corresponds to the Hamming weight of the input difference.

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Improving this bound

Let α be an input difference into f_S. For the differential probability over f_S it holds that

(1) if $\mathrm{wt}(\alpha) = 0$, then $p_\alpha = 1$ and $U_\alpha = \{0\}$

(2) if $\mathrm{wt}(\alpha) = 1$, then $p_\alpha \leq 2^{-2}$

(3) if $\mathrm{wt}(\alpha) \in \{2, 3\}$, then $p_\alpha \leq 2^{-3}$

(4) if $\mathrm{wt}(\alpha) \geq 4$, then $p_\alpha \leq 2^{-4}$

Proof.

Construct enough linearly independent elements U_α. $\square$

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Proof.

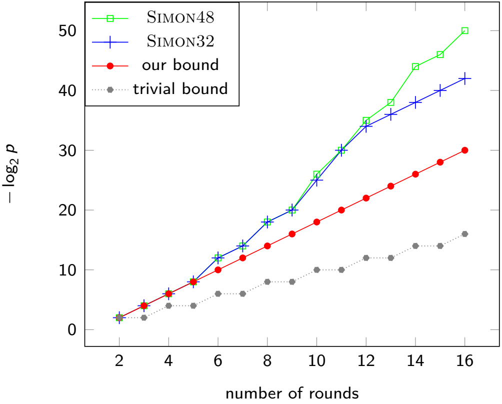
Construct enough linearly independent elements U_α. □

A Trivial Upper Bound on the Trail Probability

Worst case: The input difference into f_S of every second round is 0.

$$(0, \alpha) \to (\alpha, 0) \to (0, \alpha) \to \dots$$

If $p_\alpha = 2^{-2}$, we would obtain the *trivial bound*.



Obtaining Our Bound

For analyzing multiple rounds through the Feistel construction, we consider only trails of the form $(0, \alpha) \to \cdots \to (0, \beta)$

Observation

Let for all differences $\alpha, \beta \in \mathbb{F}_2^n \setminus \{0\}$ and all $t > 1$ the differential probability of any t-round $(0, \alpha) \to \cdots \to (0, \beta)$ trail be bounded by 2^{-2t}. Then,

$$P((\gamma_0, \delta_0) \xrightarrow{1} \ldots \xrightarrow{T} (\gamma_T, \delta_T)) \leq 2^{-2T+2}$$

for all γ_i, δ_i with $(\gamma_0, \delta_0) \neq (0, 0)$ and all $T > 0$.

Obtaining Our Bound

It is left to show that the probability of all t-round trails of the form

$$(0, \alpha) \rightarrow (\alpha, 0) \rightarrow (\gamma_2, \delta_2) \rightarrow \cdots \rightarrow (\gamma_{t-1}, \delta_{t-1}) \rightarrow (0, \beta)$$

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Note that $p_0 = 1$, $p_\alpha \leq 2^{-2}$ and $\forall \gamma_i : p_{\gamma_i} \leq 2^{-2}$. Thus, one only has to make sure to *gain* a factor of 2^{-2} which is lost in the propagation of the 0-difference.

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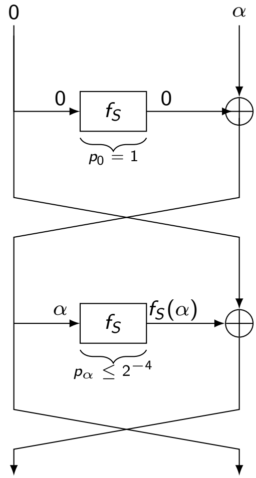
We consider serveral cases for the Hamming Weight of α.

Obtaining Our Bound

- $\mathrm{wt}(\alpha) \geq 4$:



Obtaining Our Bound $[(x \ggg 8) \wedge (x \ggg 1) \oplus (x \ggg 2)]$

- $\mathrm{wt}(\alpha) = 1$: Let w.l.o.g $\alpha = (1, 0, \ldots, 0)$.

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- $\mathrm{wt}(\alpha) = 1$: Let w.l.o.g $\alpha = (1, 0, \ldots, 0)$. Now,

$$\gamma_2 = f_S(\alpha) \oplus 0 \quad = (0, *_1, 1, 0, \ \ 0, 0, 0, 0, \ \ *_2, 0, 0, 0, \ \ 0, 0, 0, 0)$$

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If now the weight of γ_4 is higher than 1, then $p_{\gamma_3}, p_{\gamma_4} \leq 2^{-3}$. Thus, let $\mathrm{wt}(\gamma_4) = 1$.

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If now the weight of γ_4 is higher than 1, then $p_{\gamma_3}, p_{\gamma_4} \leq 2^{-3}$. Thus, let $\mathrm{wt}(\gamma_4) = 1$. It follows that

$$\gamma_5 = f_S(\gamma_4) \oplus \gamma_3 \quad = (1, 0, *, *, \;\; 1, 0, 0, *, \;\; 1, *, *, 0, \;\; 0, 0, *, 0)$$

and thus $p_{\gamma_5} \leq 2^{-3}$.

Obtaining Our Bound $[(x \ggg 8) \wedge (x \ggg 1) \oplus (x \ggg 2)]$

Case 2 ($*_2 = 1$):

Obtaining Our Bound $[(x \lll 8) \wedge (x \lll 1) \oplus (x \lll 2)]$

Case 2 ($*_2 = 1$): Then $p_{\gamma_2} \leq 2^{-3}$ already holds and

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Again, let w.l.o.g $\mathrm{wt}(\gamma_3) = 1$. It follows that

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and thus $p_{\gamma_4} \leq 2^{-3}$.

All in all, we "gained" a factor of $2^{-1} \cdot 2^{-1} = 2^{-2}$.

Obtaining Our Bound

For the cases

- $\mathrm{wt}(\alpha) = 2$
- $\mathrm{wt}(\alpha) = 3$

this can be proven in a similar way!

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Conclusion

- We took a further step into understanding possible block cipher constructions.
- For SIMON, we were able to obtain a non-trivial upper bound on the max. probability of a differential trail using a non-experimental argument.
- One can do the analysis for other rotation constants as well. Same bound is also valid for SIMECK.[3]

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Note

We did not show improved security of SIMON. Instead, we tried to learn more about possible block cipher constructions!

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Thanks for your attention!
Any Questions?